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Diffusion and first-order chemical reaction on impulsively started infinite vertical plate with variable temperature

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Abstract

An exact solution to the problem of flow past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion is presented here, taking into account the homogeneous chemical reaction of first-order. The dimensionless governing equations are solved using Laplace-transform technique. The velocity, temperature and concentration profiles are shown on graphs. It is observed that the velocity and concentration increases during generative reaction and decreases in destructive reaction.

Keywords: Heat and mass transfer; Chemical reaction; Homogeneous; Exact; Vertical plate; Variable temperature

1. Introduction

Chemical reactions can be codified as either hetrogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to concentration itself (Cussler [1]). A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role in chemical process industries such as food processing and polymer production. For example, formation of smog is a first order homogeneous chemical reaction. Consider the emission of NO_2 from automobiles and other smoke-stacks. This NO_2 reacts chemically in the atmosphere with unburned hydrocarbons (aided by sunlight) and produces Peroxyacetylnitrate, which forms an envelop of what is termed as photochemical smog.

Sakiadis [2,3] studied the growth of the two-dimensional velocity boundary layer over a continuously moving horizontal plate, emerging from a wide slot, at uniform velocity. Soundalgekar [4] has studied mass transfer effects on flow past an impulsively started infinite isothermal vertical plate. Again, Soundalgekar et al. [5] analysed the mass transfer effects on impulsively started infinite vertical plate with variable temperature or uniform heat flux. Chambre and Young [6] have analysed a first order chemical reaction in the neighbourhood of a horizontal plate. Das et al. [7] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

It is proposed to study the flow past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion in the presence a homogeneous chemical reaction of first-order. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2. Analysis

Here the flow of a viscous incompressible fluid past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion is considered.

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Nomenclature

A	constant	
C'	species concentration in the fluid	$\mathrm{mol\cdot m^{-3}}$
C	dimensionless concentration	
C_p	specific heat at constant pressure	$\mathrm{J\cdot kg^{-1}\cdot K}$
D	mass diffusion coefficient	$\mathrm{m^2\cdot s^{-1}}$
Gm	mass Grashof number	
Gr	thermal Grashof number	
g	accelaration due to gravity	$\mathrm{m\cdot s^{-2}}$
k	thermal conductivity	$\mathrm{W\cdot m^{-1}\cdot K}$
K_l	chemical reaction parameter	J
K	dimensionless chemical reaction parameter	
N	buoyancy ratio parameter	
Pr	Prandtl number	
Sc	Schmidt number	
T'	temperature of the fluid near the plate	K
t'	time	s
t	dimensionless time	
u'	velocity of the fluid in the x'-direction	$\mathrm{m\cdot s^{-1}}$
u_0	velocity of the plate	$\mathrm{m\cdot s^{-1}}$
u	dimensionless velocity	
y'	coordinate axis normal to the plate	m
y	dimensionless coordinate axis normal to the plate	

Greek symbols

α	thermal diffusivity	$\mathrm{m^2\cdot s^{-1}}$
β	volumetric coefficient of thermal expansion	$\mathrm{K^{-1}}$
β^*	volumetric coefficient of expansion with concentration	$\mathrm{K^{-1}}$
μ	coefficient of viscosity	$\mathrm{Pa\cdot s}$
ν	kinematic viscosity	$\mathrm{m^2\cdot s^{-1}}$
ρ	density of the fluid	
τ	dimensionless skin-friction	
θ	dimensionless temperature	
η	similarity parameter	
$erfc$	complementary error function	

Subscripts

w	conditions at the wall
∞	conditions in the free stream

It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Assuming no leading edge or parallel flow, so that all the convective terms are zero. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate. At time $t' \leqslant 0$, the plate and fluid are at the same temperature T'_∞ and concentration C'_∞. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against the gravitational field with uniform velocity u_0, the plate temperature is made to raise linearly with time. Also the level of the species concentration is raised to C'_w. Then by usual Boussinesq's and boundary layer approximation. The unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + \nu\frac{\partial^2 u'}{\partial y'^2} \tag{1}$$

$$\rho C_p\frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_l C' \tag{3}$$

with the following initial and boundary conditions:

$$\begin{aligned} &u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{for all } y', t' \leqslant 0\\ &t' > 0{:}\ \ u' = u_0, \quad T' = T'_\infty + \left(T'_\mathrm{w} - T'_\infty\right)At'\\ &\qquad C' = C'_\mathrm{w} \quad \text{at } y' = 0\\ &u' = 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \quad \text{as } y' \to \infty \end{aligned} \tag{4}$$

where $A = u_0^2/\nu$.

On introducing the following non-dimensional quantities:

$$\begin{aligned} &u = \frac{u'}{u_0\, Gr}, \qquad t = \frac{t' u_0^2}{\nu}, \qquad y = \frac{y' u_0}{\nu}\\ &\theta = \frac{T' - T'_\infty}{T'_\mathrm{w} - T'_\infty}, \qquad Gr = \frac{g\beta\nu(T'_\mathrm{w} - T'_\infty)}{u_0^3}\\ &C = \frac{C' - C'_\infty}{C'_\mathrm{w} - C'_\infty}, \qquad Gm = \frac{\nu g\beta^*(C'_\mathrm{w} - C'_\infty)}{u_0^3}\\ &Pr = \frac{\mu C_p}{k}, \qquad Sc = \frac{\nu}{D}, \qquad N = \frac{Gm}{Gr}, \qquad K = \frac{\nu K_l}{u_0^2} \end{aligned} \tag{5}$$

in Eqs. (1)–(4), leads to

$$\frac{\partial u}{\partial t} = \theta + NC + \frac{\partial^2 u}{\partial y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - KC \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} &u = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } y, t \leqslant 0\\ &t > 0{:}\ \ u = \frac{1}{Gr}, \quad \theta = \frac{A\nu}{u_0^2}t = A_1 t, \quad C = 1\\ &\quad \text{at } y = 0\\ &u = 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as } y \to \infty \end{aligned} \tag{9}$$

where $A_1 = (A\nu)/u_0^2$.

The numerical calculations are valid for $A_1 = 1$. Eqs. (6)–(8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[\left(1+2\eta^2 Pr\right)erfc\left(\eta\sqrt{Pr}\right) - \frac{2}{\sqrt{\pi}}\eta\sqrt{Pr}\exp\left(-\eta^2 Pr\right)\right] \tag{10}$$

$$\begin{aligned} u = {} & \frac{1}{Gr}erfc(\eta) + \frac{t^2}{6(Pr-1)} \\ & \times\left[\left(3+12\eta^2+4\eta^4\right)erfc(\eta) - \frac{\eta}{\sqrt{\pi}}\left(10+4\eta^2\right)\exp\left(-\eta^2\right) \right. \\ & - \left(3+12\eta^2 Pr + 4\eta^4 (Pr)^2\right)erfc\left(\eta\sqrt{Pr}\right) \\ & \left. + \frac{\eta\sqrt{Pr}}{\sqrt{\pi}}\left(10+4\eta^2 Pr\right)\exp\left(-\eta^2 Pr\right)\right] \\ & + \frac{N}{K\,Sc}erfc(\eta) + \frac{N\exp(at)}{2K\,Sc} \\ & \times\left[\exp\left(2\eta\sqrt{bt\,Sc}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{bt}\right) + \exp\left(-2\eta\sqrt{bt\,Sc}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{bt}\right) \right. \\ & - \exp\left(2\eta\sqrt{at}\right)erfc\left(\eta+\sqrt{at}\right) \\ & \left. - \exp\left(-2\eta\sqrt{at}\,erfc\left(\eta-\sqrt{at}\right)\right]\right. \\ & - \frac{N}{2K\,Sc}\left[\exp\left(2\eta\sqrt{Kt\,Sc}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right) \right. \\ & \left. + \exp\left(-2\eta\sqrt{Kt\,Sc}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)\right] \end{aligned} \tag{11}$$

$$C = \frac{1}{2}\left[\exp\left(2\eta\sqrt{Kt\,Sc}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Kt\,Sc}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)\right] \tag{12}$$

where,

$$\eta = y/2\sqrt{t}, \qquad a = \frac{K\,Sc}{(1-Sc)} \quad \text{and} \quad b = \frac{K(1+Sc)}{(1-Sc)}.$$

The mass diffusion Eq. (8) can be adjusted to meet these circumstances if one takes

(i) $K > 0$ for the destructive reaction,
(ii) $K = 0$ for no reaction, and
(iii) $K < 0$ for the genearative reaction.

The purpose of the calculations given here is to assess the effects of the parameters *Sc*, *N* and *K* upon the nature of the flow and transport. The numerical values of the velocity, temperature and concentration are computed for different parameters like Schmidt number, buoyancy ratio and chemical reaction parameter. The temperature profiles are calculated from Eq. (10) and these are shown in Fig. 1, for air and water. It is observed that the temperature increases with increasing the time. The effect of the Prandtl number is very important in temperature field. There is a fall in temperature due to increasing values of the Prandtl number.

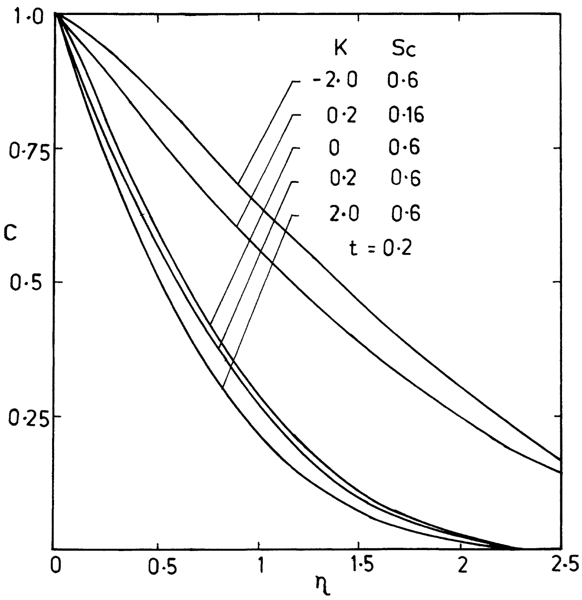

Fig. 1. Concentration profiles.

The numerical values of the concentration profiles are computed from Eq. (12) and these values are plotted in Fig. 1. for different values of the Schmidt number and chemical reaction parameter. The effect of Chemical reaction parameter and Schmidt number are very important in concentration field. Chemical reaction increases the rate of interfacial mass transfer. The reaction reduces the local concentration, thus increasing its concentration gradient and its flux. It is observed that the concentration increases during generative reaction and decreases in destructive reaction. The concentration increases with decreasing Schmidt number. It is clear that, the concentration increases tremendously in the presence of geneartive reaction.

The effects of buoyancy ratio parameter for both aiding ($N > 0$) as well as opposing ($N < 0$) are shown in Fig. 2. It is observed that the velocity increases in the presence of opposing flows and decreases with aiding flows. This trend is just reversed in the case of stationary vertical plate (Gebhart and Pera [8]).

The velocity profiles for different values of the Schmidt number, chemical reaction parameter and the time are shown in Fig. 3. It is clear that the velocity increases with increasing chemical reaction parameter or time. It is also observed that the velocity decreases with increasing values of the Schmidt number.

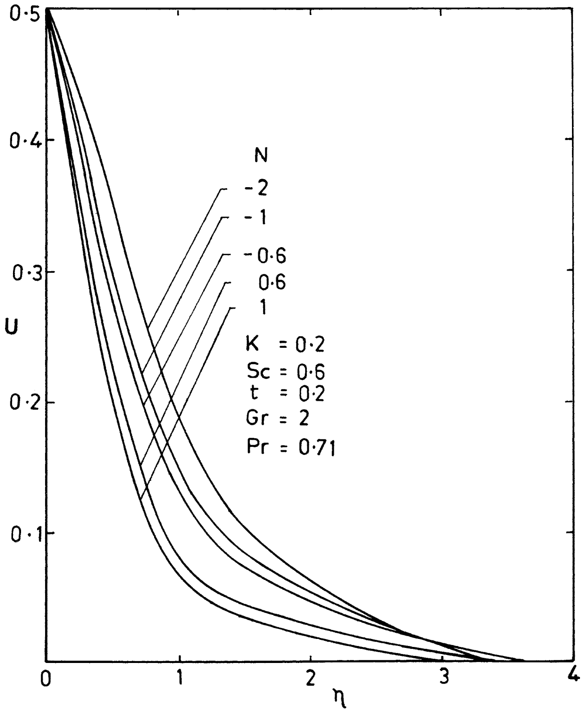

Fig. 2. Velocity profiles for different N.

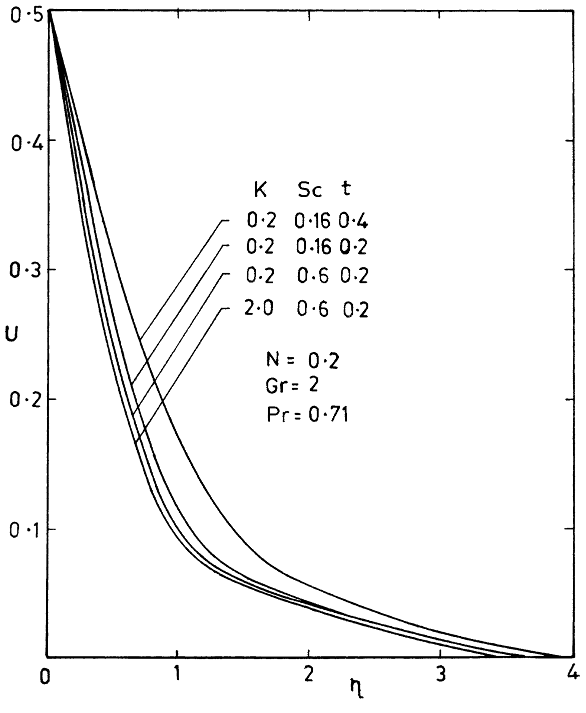

Fig. 3. Velocity profiles for different K, Sc and t.

From the velocity field, the efect of mass transfer on the skin-friction is studied and is given in dimensionless from as

$$\tau = -\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)_{\eta=0} \tag{13}$$

Hence, from Eqs. (11) and (13),

Table 1
Values of the skin-friction τ

t	Sc	N	K	$Pr = 0.71$	$Pr = 7.0$
0.2	0.6	0.2	0.2	2.8929	2.9109
0.2	0.6	−0.2	0.2	1.7043	1.6863
0.2	0.16	0.2	0.2	8.7652	8.7833
0.2	0.6	0	0.2	0.5943	0.6123
0.2	0.6	0.2	−0.2	1.9410	1.9236
0.4	0.6	0.2	0.2	2.1021	2.1532

$$\begin{aligned}\tau = {} & \frac{1}{\sqrt{\pi t}}\left[\frac{1}{Gr} - \frac{4}{3}\frac{t^2}{(\sqrt{Pr}+1)} + \frac{N}{K\,Sc}\right] \\ & + \frac{N\exp(at)}{K\,Sc}\left[\sqrt{Sc}\,\mathrm{erf}(\sqrt{bt}) - \sqrt{a}\,\mathrm{erf}(\sqrt{at})\right] \\ & - \frac{N}{K\,Sc}\mathrm{erf}(\sqrt{Kt})\end{aligned} \tag{14}$$

The numerical values of τ are computed and listed in the following table for $Gr = 2$. It is observed from the table, that an increase in the Schmidt number or time leads to fall in skin-friction. It is observed that the skin-friction increases in the presence of aiding flows and decreases in the presence of opposing flow. Moreover, the value of the skin-friction increases in the presence of destructive reaction and decreases with generative reaction. In a frame of reference moving with the plate with velocity u_0 upwards, the plate is a rest and the free stream has a uniform velocity downwards. Thus in this case the free stream velocity is opposite to the direction of bouyancy force which acts vertically upwards. This leads to flow separation. It is also observed that the skin-friction is more in water as compared to that in air. But, this trend is just reversed in the presence of generative reaction or opposing flow.

3. Conclusions

An exact analysis is performed to study the flow past an impulsively started infinite vertical plate in the presence of variable temperature and uniform mass diffusion. A homogeneous first-order chemical reaction between the fluid and the species concentration. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like Schmidt number, buoyancy ratio and chemical reaction parameter are studied. It is observed that the velocity increases due to the presence of the foreign mass. Conclusions of the study are as follows:

(i) The velocity and concentration increases during generative reaction ($K < 0$) and decreases in destructive reaction ($K > 0$).
(ii) The velocity increases tremendously in the presence of generative reaction.
(iii) The velocity increases in the presence of opposing flows ($N < 0$) and decreases with aiding flows ($N > 0$).

(iv) The skin-friction increases during destructive reaction and decreases in generative reaction.

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